Conversion to Jordan-canonical form

(derived here only for the case with n single eigenvalues).

[The general case will be treated in ECE 501.]

Let us look once more at the defining equation for eigen=values and eigenvectors:

$$A \cdot \underline{v}_i = \lambda_i \cdot \underline{v}_i \quad ; \; i \in 1..n$$

We can take all these vector equations, and rewrite them as a single matrix equation:

$$\underbrace{\left[A\cdot\underline{v}_1 \,\vdots\, A\cdot\underline{v}_2 \,\vdots\, \cdots \,\vdots\, A\cdot\underline{v}_n \right]}_{A \cdot \left[\underline{v}_1 \,\vdots\, \underline{v}_2 \,\vdots\, \cdots \,\vdots\, \underline{v}_n \right]} = \left[\lambda_1 \underline{v}_1 \,\vdots\, \lambda_2 \underline{v}_2 \,\vdots\, \cdots \,\vdots\, \lambda_n \underline{v}_n \right]$$

Let:

$$V = \left[\underline{v}_1 \,\vdots\, \underline{v}_2 \,\vdots\, \cdots \,\vdots\, \underline{v}_n \right]$$

be the (right) modal matrix associated with the matrix A. It is the matrix that consists of the concatenation of the (right) eigenvectors, $\underline{v}_i$, of A.

$$\Rightarrow \quad A \cdot V = \left[\lambda_1 \underline{v}_1 \,\vdots\, \lambda_2 \underline{v}_2 \,\vdots\, \cdots \,\vdots\, \lambda_n \underline{v}_n \right]$$

e.g. (n=3):

$$\left[\lambda_1 \underline{v}_1 \,\vdots\, \lambda_2 \underline{v}_2 \,\vdots\, \lambda_3 \underline{v}_3 \right] = \begin{bmatrix} \lambda_1 v_{11} & \lambda_2 v_{12} & \lambda_3 v_{13} \\ \lambda_1 v_{21} & \lambda_2 v_{22} & \lambda_3 v_{23} \\ \lambda_1 v_{31} & \lambda_2 v_{32} & \lambda_3 v_{33} \end{bmatrix}$$

$$\equiv \underbrace{\begin{bmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \\ v_{31} & v_{32} & v_{33} \end{bmatrix}}_{V} \cdot \underbrace{\begin{bmatrix} \lambda_1 & \phi & \phi \\ \phi & \lambda_2 & \phi \\ \phi & \phi & \lambda_3 \end{bmatrix}}_{\Lambda}$$

The eigenvalue/eigenvector equations can be written in a more compact form as:

$$\boxed{A \cdot V = V \cdot \Lambda}$$

V : right modal matrix of A

Λ : eigenvalue matrix of A.

$$\Lambda = \begin{bmatrix} \lambda_1 & & & \emptyset \\ & \lambda_2 & & \\ & & \ddots & \\ \emptyset & & & \lambda_n \end{bmatrix}$$

is the system matrix in Jordan-canonical form after the transformation.

$$\Rightarrow \hat{A} = \Lambda = V^{-1} \cdot A \cdot V$$

$$\equiv T \cdot A \cdot T^{-1}$$

$$\Rightarrow \boxed{T = V^{-1}}$$

Using the inverse of the right modal matrix in a similarity transformation converts an arbitrary state-space description with n distinct eigenvalues to the diagonal (Jordan) form.

The normalization of the eigenvectors only influences the $\underline{b}$ and $\underline{c}'$ vectors.

$$\dot{\underline{x}} = \begin{bmatrix} \lambda_1 & & & \emptyset \\ & \lambda_2 & & \\ & & \ddots & \\ \emptyset & & & \lambda_n \end{bmatrix} \underline{x} + \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} u$$

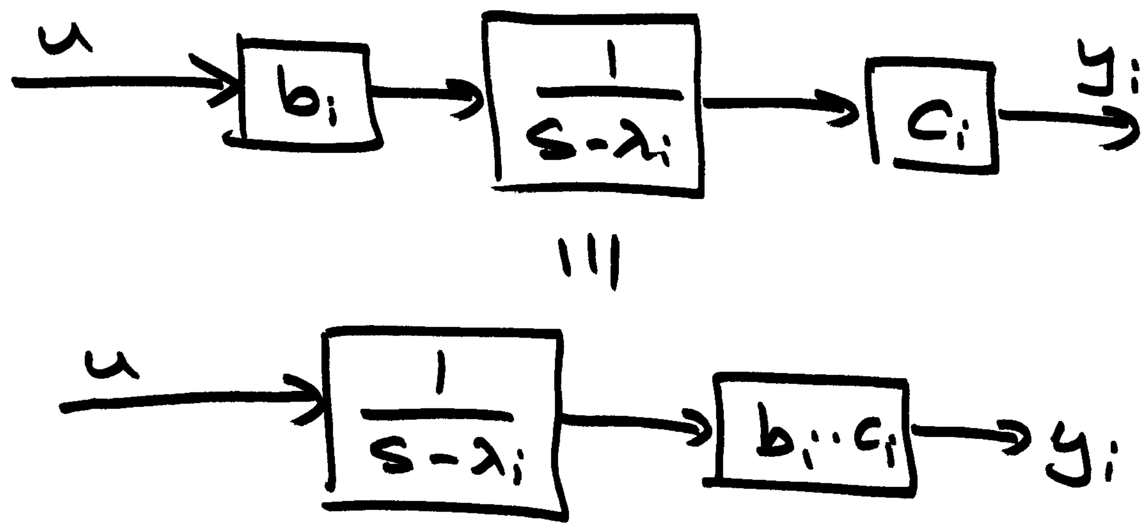
$$y = [c_1 \, c_2 \dots c_n] \, \underline{x}$$

$$\Rightarrow \dot{x}_i = \lambda_i x_i + b_i u$$

$$y_i = c_i \cdot x_i$$

$$y = \sum_{\forall i=1}^{n} y_i$$

Block diagram:



We can easily normalize the b_i to 1, by multiplying $b_i \cdot c_i \to \hat{c}_i$.

This works, unless $b_i = \emptyset$ [cf. ECE 441].

In Matlab:

$$[V, \Lambda] = eig(A)$$

$Ah = \Lambda$

$bh = V \backslash b$

$ch = c * V$

$chh = ch .* conj(bh')$

$bhh = ones(size(bh))$

$(T = inv(V))$

$= T \cdot A \cdot T^{-1}$

$= T \cdot b$

$= c / T$