Introduction

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A large majority of *engineering systems exhibit many discontinuities* that need to be included in their models.

The numerical integration methods for dynamic systems that we developed until now can thus not be used for the simulation of *hybrid systems*. We shall need something better.
Abusing the Step-size Control

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Consequently, the algorithm reduces the step size in order to capture this eigenvalue in its accuracy domain.

However, the new “eigenvalue” is a joker. It doesn’t allow itself to be captured. Irrespective of how much the step size is being reduced, the eigenvalue remains outside the accuracy domain.
Abusing the Step-size Control II

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Once the discontinuity lies in the past, the evasive eigenvalue disappears as miraculously as it had shown up before.

The step-size control algorithm slowly increases the step size again, until it reaches its optimal value . . . or until it encounters the next discontinuity, whichever happens first.
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However, this approach is neither efficient nor robust. Sometimes, it fails miserably.
Swiss trains are running on AC voltage at $16\frac{2}{3}$ Hz.
Speed Control in Train Engines

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The most simple speed control circuit is shown to the left. The resistor with an inductor in series represents the load (the train). The thyristor blocks negative current (operating as a diode) and also blocks an additional part of every period.
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The percentage of the period that is not being blocked is controlled by the firing angle of the thyristor.
Speed Control in Train Engines II

Thyristor-controlled Train Engine

- Voltage $V_{Load}$ [V]
- Current $i_{Load}$ [A]

Graphs showing the variation of voltage and current over time.
The simulation trajectories exhibit discontinuities.
Speed Control in Train Engines II

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- The electric power \( P = v_{\text{Load}} \cdot i_{\text{Load}} \) depends on the firing angle.
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For this simulation, we used a firing angle of 30°. The speed of the train gets reduced with larger firing angles.
Speed Control in Train Engines III

Power Spectrum of Thyristor-controlled Engine

- Frequency [Hz]
- Power Spectrum

![Graph showing the power spectrum of a thyristor-controlled engine.](image-url)
There is much power contained in the *third harmonic* at 50Hz.
There is much power contained in the third harmonic at 50Hz.

In the past, when trains with three thyristor-controlled locomotives slowly climbed up the St. Gotthard mountain, the electric counters of the houses near to the rails were reset to zero, to everyone’s content ... except for the local power company of the Canton of Uri.
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This control strategy is called *burst control strategy*.

The trains made use of *packets of eight periods*. 
Speed Control in Train Engines V

Burst-controlled Train Engine

\[ V_{\text{Load}} [V] \]

\[ i_{\text{Load}} [A] \]

\[ \text{time [sec]} \]
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When the trains departed from the stations, the passengers noticed the abrupt velocity changes and received a free massage of their stomach muscles.
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There is power in the higher harmonics at $4\frac{1}{6}$ Hz and at $6\frac{1}{4}$ Hz, but nothing at 50 Hz. Hence there is no problem with cross-talk on the electric commercial grid.
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To avoid the problem with the discrete velocities, the burst would have to be made longer, e.g. including 16 or even 32 periods. However, this is not possible either, as the periodicity of the signal would then shrink further. The lowest periodicity that is allowed for security reasons is 2 Hz, because otherwise, the trains would not react fast enough, e.g. when passing a closed semaphore.
To avoid the problems encountered before, more advanced control circuits were developed that made use of a *four-quadrant rectifier* with *commutation*.
The line current, $i_L$, is controlled in such a way that it always remains in the vicinity of

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Speed Control in Train Engines VIII

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At that moment, $A_z$ assumes a value of $A_z = 1.0$, and $i_L$ decays quickly again until it reaches $(Y - B_T)$, where $A_z$ takes a value of $A_Z = 0.0$ as before.
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$$B_T = 200.0 \text{ Amps}$$

is the allowed tolerance around $Y(t)$, within which $i_L$ is supposed to operate.
We expect to obtain the following trajectories from the simulation:

![SCR-controlled Train Engine](image_url)
Speed Control in Train Engines IX

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- The trajectories exhibit very serious discontinuities. For this reason, the simulation requires a *highly efficient and accurate step-size control*.

- These signals contain much power at high frequencies in their power spectrum. Yet, these contributions are at frequencies much higher than 50Hz.

- Thus, we don’t experience problems with cross-talk.
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During some time interval, the *simulation trajectories obtained were incorrect*. During those periods, *the simulation was creeping along* using the *smallest integration step size permitted by the software*.
What happened?
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- We used an explicit RK4 algorithm with abuse of step-size control for handling the discontinuities.
Speed Control in Train Engines XI

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- It was an algorithm in four stages.
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- If the number of commutations within a step was *even*, we ended up at the end of the step on the same side of the fence and had to try jumping over the fence once again during the next step.
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- If the number of commutations within a step was even, we ended up at the end of the step on the same side of the fence and had to try jumping over the fence once again during the next step.
- The simulation proceeded during much time with the smallest allowed step size, unable to cross the fence.
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▶ If the number of commutations within a step was *even*, we ended up at the end of the step on the same side of the fence and had to try jumping over the fence once again during the next step.

▶ The simulation proceeded during much time with the smallest allowed step size, unable to cross the fence.

Abuse of integration step-size control for the handling of discontinuities is not a good idea. The resulting algorithms aren’t robust. Sometimes, the technique works quite well, but at other times, it fails miserably.
Our problems arose from the fact that \textit{we didn't instruct} the ODE solver that there were discontinuities. The ODE solver is incapable of \textit{reading and interpreting} models. It can only \textit{execute} them.
Time Events

Our problems arose from the fact that *we didn't instruct* the ODE solver that there were discontinuities. The ODE solver is incapable of *reading and interpreting* models. It can only *execute* them.

What we need is a *syntactical element* in the *model description language* that enables us to explicitly inform the integrator of occurrences of discontinuities.
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From now on, we shall call discontinuities discrete events. What we need are explicit mechanisms for the description and handling of events.
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From now on, we shall call discontinuities discrete events. What we need are explicit mechanisms for the description and handling of events.

In some cases, the time of occurrence of an event is known in advance. In this case, we talk about time events.
Let us consider once more the train speed control by a single thyristor. The time when the thyristor closes is known beforehand. The thyristor closes for the first time $\alpha$ degrees after the beginning of the period.
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We can calculate:

\[
\begin{align*}
\Delta t_{\text{period}} &= \frac{1}{2\pi f} \\
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The first occurrence of this event can be *planned ahead*. Thus, this is a time event.

One of the actions associated with the closing event is the planning (scheduling) of the next occurrence of the same event at time $t + \Delta t_{\text{period}}$, where $t$ denotes the current time.
One way to advise the simulation of forthcoming time events is by offering in the language a *schedule* statement that can be used to schedule future time events.
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For example, we might include in the *initial section* of the program the following statements:

```
Gate = open;
schedule CloseGateEvent at Δt_{event};
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and in the code of the `CloseGateEvent function`:

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schedule CloseGateEvent at t + Δt_{period};
```

The event, during which the thyristor opens again, is a different type of event. We shall talk about that class of events later.
It should be mentioned that the *discontinuity associated with the event* is not part of the continuous-time model. All simulation trajectories in between events are perfectly continuous. Only the *conditions under which events are to occur* are known during the continuous simulation.
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The *localization of a time event* is trivial. All we need in order to localize time events accurately is an ODE solver that provides *dense output*. We proceed in exactly the same manner that we use to localize a communication instant, at which we wish to *report the values of the output variables*. 
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If the integration algorithm reduces the step in order to hit the \textit{communication instants} (common for single-step algorithms), we shall do the same in order to arrive at the time of the next time event.
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On the other hand, if the integration algorithm uses interpolation for the purpose of calculating the values of the output variables at communication instants (common for multi-step algorithms), we shall do the same for localizing time events.
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Afterwards, we are dealing with a new continuous-time simulation with new initial conditions.
Time Events V

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The *time events* are stored in an *event calendar*, a linear linked list, in such a way that the *next time event* is always stored at the beginning of the list.
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The continuous simulation only needs to know the *instant of time when the next time event is scheduled to occur*.

The continuous simulation proceeds until the time of occurrence of the next event. At that moment, the continuous simulation terminates, the actions associated with the event (i.e., the discontinuity) are being processed, and a new continuous simulation starts with new initial conditions.
Consequently, the continuous simulations are now executed in segments. The individual continuous-time simulation segments are interrupted for handling discrete events that occur.
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The simulation of a hybrid model is called *hybrid simulation*.
Digital control systems can be modeled by means of hybrid models. They can then be simulated using a hybrid simulation engine.
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- Stabilizing, Linearizing and Decoupling Controller
- Trajectory Planner
- Path Planner
- Task Planner
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- There are four closed loops employing different sampling rates $T_1 \leq T_2 \leq T_3 \ll T_4$.
- Every sampler represents an infinite series of self-scheduling time events.
- The event calendar maintains the four types of events and decides at every moment, which is the next event that needs to be executed.
State Events

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In the thyristor control example, the gate opening event is a state event. We don’t know ahead of time, when it happens. We only know under what conditions it happens, i.e., when the load current passes through zero.
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In the thyristor control example, the gate opening event is a state event. We don’t know ahead of time, when it happens. We only know under what conditions it happens, i.e., when the load current passes through zero.

To this end, we might include in the section where the simulation equations are being described the following statement:

```
schedule OpenGateEvent when i_Load < 0;
```

and the code of the OpenGateEvent function consists of:

```
Gate = open;
```
State Events II

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During the continuous simulation, all active event detection functions must be constantly monitored to check whether any of the zero crossings has taken place.
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- During the continuous simulation, all active event detection functions must be constantly monitored to check whether any of the zero crossings has taken place.

- Once one of the event detection functions has found a zero crossing, an *event localization* algorithm is triggered that iterates on the exact time of the zero crossing.

- In the mathematical literature, the event localization algorithm is often also referred to as *root finding algorithm*. 
Multiple Zero Crossings

If more than one event detection function is triggered during a single integration step, we need an algorithm to determine, which of the zero crossing happens first.
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The next evaluation time instant is calculated using the formula:

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t_{\text{next}} = \min_{\forall i} \left\{ \frac{f_i(t_{k+1}) \cdot t_k - f_i(t_k) \cdot t_{k+1}}{f_i(t_{k+1}) - f_i(t_k)} \right\}
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- On the other hand, if there are multiple zero crossings in the interval \( t \in [t_k, t_{\text{next}}] \), we set \( t_{k+1} = t_{\text{next}} \) and repeat the algorithm.

---

**Numerical Simulation of Dynamic Systems XXI**

- Simulation of Discontinuous Systems
- State Events

**Multiple Zero Crossings**
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- If there is exactly one zero crossing in the interval \( t \in [t_k, t_{\text{next}}] \), we simplified the problem to that of localizing a single zero crossing.
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Figure: Golden Section.
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$$\frac{x}{1} = \frac{1-x}{x} \Rightarrow x = 0.618$$
How can the *golden section method* be used for *event isolation*?

---

**Iteration of State Events by Golden Section**

Zero-crossing Functions

- $f_1$
- $f_2$
- $f_3$

Zero-crossing Functions

- $f_1$
- $f_2$
- $f_3$

*time [sec]*

$t_k$, $t_k + (1 - x)h$, $t_k + xh$, $t_{k+1}$
How can the *golden section method* be used for *event isolation*?

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- If there is no zero crossing in the interval \( t \in [t_k, t_k + (1 - x) \cdot h] \), we set \( t_k = t_k + (1 - x) \cdot h \) and repeat the algorithm with one new intermediate point.
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- On the other hand, if there are multiple zero crossings in the interval \( t \in [t_k, t_k + (1 - x) \cdot h] \), we set \( t_{k+1} = t_k + x \cdot h \) and repeat the algorithm with one new intermediate point.
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If more than one zero crossing occurs within a single integration step, we reduce the interval \( t \in [t_k, t_{k+1}] \) using either one of the two proposed methods: *Regula Falsi* or *golden section*, until there remains exactly one *zero crossing* of a single *event detection function* within the time interval.
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It can of course happen that two separate zero crossings occur at exactly the same time instant. In that case, the event isolation algorithm continues until the time interval has been sufficiently reduced so that we can consider the event localized, i.e., the *event isolation* algorithm is then also used for *event localization*. 
We have meanwhile solved the problem of event isolation.

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Thus, the original problem has thus been simplified to the problem of localizing a single event that has already been identified to occur within a given fixed time interval.
Event Localization

Single-step Algorithms

Evidently, the *event isolation* techniques discussed previously can also be used for *event localization*.
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They may, however, be unnecessarily inefficient due to their *linear convergence speed*.
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We could suspect that Newton iteration might work better due to the quadratic convergence speed of this method.
Unfortunately, Newton iteration does not always converge. In the given example, if we start from the right, we obtain quick convergence, whereas if we start from the left, already the next iteration step carries us outside the interval.
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As we already know that the zero crossing occurs within a defined time interval, we are not interested in ever leaving the interval in search of the zero crossing.
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If the next Newton iteration step carries us outside the interval when starting on both ends of the interval, it may be advisable to reduce the width of the interval a bit more using one of the two event isolation algorithms introduced earlier before switching to Newton iteration for increasing the convergence speed.
Unfortunately, Newton iteration does not always converge. In the given example, if we start from the right, we obtain quick convergence, whereas if we start from the left, already the next iteration step carries us outside the interval.

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If the next Newton iteration step carries us outside the interval when starting on both ends of the interval, it may be advisable to reduce the width of the interval a bit more using one of the two event isolation algorithms introduced earlier before switching to Newton iteration for increasing the convergence speed.

Once the time interval has been sufficiently reduced, Newton iteration will converge, and will do so more rapidly than either Regula Falsi or golden section.
Another algorithm that is possibly even more efficient for event localization than Newton iteration is polynomial interpolation.
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As every event detection function can be expressed as a function of state variables, we know not only the values of the event detection functions at both ends of the time interval, but also their time derivatives.
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As every event detection function can be expressed as a function of state variables, we know not only the values of the event detection functions at both ends of the time interval, but also their time derivatives.

Consequently, we may propose an interpolation polynomial:

\[ p(t) = a \cdot t^3 + b \cdot t^2 + c \cdot t + d \]

with the time derivative:

\[ \dot{p}(t) = 3a \cdot t^2 + 2b \cdot t + c \]
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We know that:

\[ p(t_k) = a \cdot t_k^3 + b \cdot t_k^2 + c \cdot t_k + d = f_k \]
\[ p(t_{k+1}) = a \cdot t_{k+1}^3 + b \cdot t_{k+1}^2 + c \cdot t_{k+1} + d = f_{k+1} \]
\[ \dot{p}(t_k) = 3a \cdot t_k^2 + 2b \cdot t_k + c = \dot{f}_k = h_k \]
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Event Localization IV
Single-step Algorithms

We conclude:

\[
\begin{pmatrix}
  f_k \\
  f_{k+1} \\
  h_k \\
  h_{k+1}
\end{pmatrix} =
\begin{pmatrix}
  t_k^3 & t_k^2 & t_k & 1 \\
  t_k^3 & t_{k+1}^2 & t_{k+1} & 1 \\
  3t_k^2 & 2t_k & 1 & 0 \\
  3t_{k+1}^2 & 2t_{k+1} & 1 & 0
\end{pmatrix}
\begin{pmatrix}
  a \\
  b \\
  c \\
  d
\end{pmatrix}
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\[
\begin{pmatrix}
  f_k \\
  f_{k+1} \\
  h_k \\
  h_{k+1}
\end{pmatrix} =
\begin{pmatrix}
  t_3^k \\
  t_3^{k+1} \\
  3t_2^k \\
  3t_2^{k+1}
\end{pmatrix}
\begin{pmatrix}
  t_2^k \\
  t_2^{k+1} \\
  2t_k \\
  2t_{k+1}
\end{pmatrix}
\begin{pmatrix}
  t_k \\
  t_{k+1} \\
  1 \\
  1
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\cdot
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  a \\
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From the linear system of equations, we can easily determine the values of the four coefficients of the interpolation polynomial.
Event Localization IV

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It makes use of all of the information (the two function values and the two derivative values) available, and consequently, it is optimally suited for the task at hand.
The *Cubic interpolation* algorithm is characterized by a *cubic convergence speed*. For this reason, the algorithm converges even faster than *Newton iteration*.

It makes use of all of the information (the two function values and the two derivative values) available, and consequently, it is optimally suited for the task at hand.

Furthermore, the algorithm encounters at least one solution inside the time interval. Consequently, the *convergence can be guaranteed* just as in the case of the *Regula Falsi* and *golden section* methods.
If we are using a *multi-step algorithm* for the simulation, the event localization algorithm can be improved even further, because we have access to the *Nordsieck vector*.
Event Localization
Multi-step Algorithms

If we are using a multi-step algorithm for the simulation, the event localization algorithm can be improved even further, because we have access to the Nordsieck vector.

We can write:

\[
F(\hat{h}) = F_i(t_{\text{next}}) = F_i(t_{k+1}) + \hat{h} \frac{dF_i(t_{k+1})}{dt} + \frac{\hat{h}^2}{2} \frac{d^2F_i(t_{k+1})}{dt^2} + \frac{\hat{h}^3}{6} \frac{d^3F_i(t_{k+1})}{dt^3} + \cdots = 0.0
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- This is a function of the unknown \( \hat{h} \) that can be solved for the unknown by Newton iteration.
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This is a function of the unknown \( \hat{h} \) that can be solved for the unknown by Newton iteration.

We begin with \( \hat{h}^0 = 0.5 \cdot (t_k - t_{k+1}) \) and iterate:

\[
\hat{h}^{\ell+1} = \hat{h}^\ell - \frac{\mathcal{F}(\hat{h}^\ell)}{\mathcal{H}(\hat{h}^\ell)}
\]

where:

\[
\mathcal{H}(\hat{h}) = \frac{d\mathcal{F}(\hat{h})}{d\hat{h}} = \frac{d\mathcal{F}_i(t_{k+1})}{dt} + \hat{h} \frac{d^2\mathcal{F}_i(t_{k+1})}{dt^2} + \frac{\hat{h}^2}{2} \frac{d^3\mathcal{F}_i(t_{k+1})}{dt^3} + \cdots
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As we have access to the *Nordsieck vector*, this iteration converges in a single iteration step with the same precision as the integration itself. We couldn’t hope for anything more.
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We only have access to the Nordsieck vector for state variables. Thus, if the event detection function doesn’t coincide with one of the state variables, it may be worth our while to augment the differential equation system with:

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\dot{x}_{n+i} = \frac{dF_i(x)}{dt}
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In this way, we shall need to integrate one additional state equation, but the technique simplifies and accelerates the localization of events.
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Since event localization may occupy much of the overall simulation time when dealing with a system with frequent discontinuities, such as in the case of the last version of the train speed control, augmenting the model in this fashion may turn out to be economical.
Non-essential State Events

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- Initially, we detect the zero crossing of function $f_2$.
- While iterating on this zero crossing, a second and earlier zero crossing of function $f_1$ is detected.
- The event associated with function $f_2$ is called \textit{non-essential}, because no action is associated with this event.
Non-essential State Events

Sometimes it is a good idea to augment the set of event detection functions by an additional function that is the derivative of another.

Initially, we detect the zero crossing of function $f_2$.

While iterating on this zero crossing, a second and earlier zero crossing of function $f_1$ is detected.

The event associated with function $f_2$ is called *non-essential*, because no action is associated with this event.

However, without the additional event detection function, $f_2$, we would have missed the two essential zero crossings of function $f_1$. 
In this presentation, we have discussed why *models containing discontinuities require special provisions* both on the side of the modeling language (the discontinuities should be declared in the model either directly or indirectly) and also on the side of the simulation engine (we should not simulate across discontinuities).
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It was shown that the step-size control algorithm may allow a model with discontinuities to be simulated successfully even without offering root-solving algorithms to the simulation engine, but we also demonstrated that this approach can fail.
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We then introduced the two types of events, the *time events* and the *state events*. 
Conclusions

▶ In this presentation, we have discussed why *models containing discontinuities require special provisions* both on the side of the modeling language (the discontinuities should be declared in the model either directly or indirectly) and also on the side of the simulation engine (we should not simulate across discontinuities).

▶ It was shown that the step-size control algorithm may allow a model with discontinuities to be simulated successfully even without offering root-solving algorithms to the simulation engine, but we also demonstrated that this approach can fail.

▶ We then introduced the two types of events, the *time events* and the *state events*.

▶ In the final part of the presentation, we focused on the *root-solving algorithms* themselves. We decomposed the overall problem into an *event isolation* algorithm and an *event localization* algorithm.
References

