Numerical Simulation of Dynamic Systems: Hw9 - Solution

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Given the electrical circuit:
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The circuit contains a constant voltage source, $u_0$, and a dependent current source, $i_4$, that depends on the voltage across the capacitor, $C$, and the resistor, $R_3$. 

$$R = 100$$

$$R_1 = 100$$

$$R_2 = 100$$

$$C = 1 \times 10^{-6}$$

$$L = 0.01$$

$$v_0 = 10$$

$$i_4 = 4v_3$$
Given the electrical circuit:

- The circuit contains a constant voltage source, $u_0$, and a dependent current source, $i_4$, that depends on the voltage across the capacitor, $C$, and the resistor, $R_3$.

- Write down the element equations for the seven circuit elements. Since the voltage $u_3$ is common to two circuit elements, these equations contain 13 rather than 14 unknowns. Add the voltage equations for the three meshes and the current equations for three of the four nodes.
[H7.1] Electrical Circuit, Horizontal and Vertical Sorting II

- Draw the structure digraph of the DAE system, and apply the Tarjan algorithm to sort the equations both horizontally and vertically. Write down the causal equations, i.e., the resulting ODE system.
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Simulate the ODE system across 50 $\mu$sec using RKF4/5 with Gustaffsson step-size control and with zero initial conditions on both the capacitor and the inductor.
[H7.1] Electrical Circuit, Horizontal and Vertical Sorting II

- Draw the structure digraph of the DAE system, and apply the Tarjan algorithm to sort the equations both horizontally and vertically. Write down the causal equations, i.e., the resulting ODE system.

- Simulate the ODE system across 50 µsec using RKF4/5 with Gustaffsson step-size control and with zero initial conditions on both the capacitor and the inductor.

- Plot the voltage $u_3$ and the current $i_C$, and the step size $h$ on three separate subplots as functions of time.
[H7.1] Electrical Circuit, Horizontal and Vertical Sorting III

\[ \begin{align*}
R &= 100 \\
R_1 &= \text{Horizontal Sorting} \\
C &= 1 \times 10^{-6} \\
R_2 &= \text{Vertical Sorting} \\
R_3 &= 20 \\
L &= 0.01 \\
u_0 &= 10 \\
i_4 &= 4u_3 \\
\end{align*} \]
[H7.1] Electrical Circuit, Horizontal and Vertical Sorting III

1: \( u_0 = 10 \)
2: \( u_1 = R_1 \cdot i_1 \)
3: \( u_2 = R_2 \cdot i_2 \)
4: \( u_3 = R_3 \cdot i_3 \)
5: \( i_C = C \cdot \frac{du_3}{dt} \)
6: \( u_L = L \cdot \frac{di_L}{dt} \)
7: \( i_4 = 4 \cdot u_3 \)
8: \( u_0 = u_1 + u_3 \)
9: \( u_L = u_1 + u_2 \)
10: \( u_2 = u_3 + u_4 \)
11: \( i_0 = i_1 + i_L \)
12: \( i_1 = i_2 + i_C + i_3 \)
13: \( i_4 = i_2 + i_L \)
[H7.1] Electrical Circuit, Horizontal and Vertical Sorting IV

1: \[ u_0 = 10 \]
2: \[ u_1 = R_1 \cdot i_1 \]
3: \[ u_2 = R_2 \cdot i_2 \]
4: \[ u_3 = R_3 \cdot i_3 \]
5: \[ i_C = C \cdot \frac{du_3}{dt} \]
6: \[ u_L = L \cdot \frac{di_L}{dt} \]
7: \[ i_4 = 4 \cdot u_3 \]

8: \[ u_0 = u_1 + u_3 \]
9: \[ u_L = u_1 + u_2 \]
10: \[ u_2 = u_3 + u_4 \]

11: \[ i_0 = i_1 + i_L \]
12: \[ i_1 = i_2 + i_C + i_3 \]
13: \[ i_4 = i_2 + i_L \]
1: \( u_0 = 10 \)
2: \( u_1 = R_1 \cdot i_1 \)
3: \( u_2 = R_2 \cdot i_2 \)
4: \( u_3 = R_3 \cdot i_3 \)
5: \( i_C = C \cdot \frac{du_3}{dt} \)
6: \( u_L = L \cdot \frac{di_L}{dt} \)
7: \( i_4 = 4 \cdot u_3 \)
8: \( u_0 = u_1 + u_3 \)
9: \( u_L = u_1 + u_2 \)
10: \( u_2 = u_3 + u_4 \)
11: \( i_0 = i_1 + i_L \)
12: \( i_1 = i_2 + i_C + i_3 \)
13: \( i_4 = i_2 + i_L \)
[H7.1] Electrical Circuit, Horizontal and Vertical Sorting V
[H7.1] Electrical Circuit, Horizontal and Vertical Sorting V

\begin{align*}
\text{Eq.}(1) & : u_0 \\
\text{Eq.}(2) & : i_0 \\
\text{Eq.}(3) & : u_1 \\
\text{Eq.}(4) & : i_1 \\
\text{Eq.}(5) & : u_2 \\
\text{Eq.}(6) & : i_2 \\
\text{Eq.}(7) & : i_3 \\
\text{Eq.}(8) & : i_C \\
\text{Eq.}(9) & : u_4 \\
\text{Eq.}(10) & : u_L \\
\text{Eq.}(11) & : \frac{di_L}{dt} \\
\text{Eq.}(12) & : \frac{du_3}{dt} \\
\text{Eq.}(13) & : i_4 \\
\end{align*}
[H7.1] Electrical Circuit, Horizontal and Vertical Sorting VI

\begin{center}
\begin{tikzpicture}
\node (u0) at (0,0) {$u_0$};
\node (i0) at (0,-1) {$i_0$};
\node (u1) at (0,-2) {$u_1$};
\node (i1) at (0,-3) {$i_1$};
\node (u2) at (0,-4) {$u_2$};
\node (i2) at (0,-5) {$i_2$};
\node (iC) at (0,-6) {$i_C$};
\node (u4) at (0,-7) {$u_4$};
\node (uL) at (0,-8) {$u_L$};
\node (diL/dt) at (0,-9) {$\frac{di_L}{dt}$};
\node (du3/dt) at (0,-10) {$\frac{du_3}{dt}$};
\node (i4) at (0,-11) {$i_4$};
\node (Eq.(1)) at (-1,-1) {Eq.(1)};
\node (Eq.()) at (-1,-2) {Eq.()};
\node (Eq.()) at (-1,-3) {Eq.()};
\node (Eq.(2)) at (-1,-4) {Eq.(2)};
\node (Eq.()) at (-1,-5) {Eq.()};
\node (Eq.()) at (-1,-6) {Eq.()};
\node (Eq.(3)) at (-1,-7) {Eq.(3)};
\node (Eq.()) at (-1,-8) {Eq.()};
\node (Eq.()) at (-1,-9) {Eq.()};
\node (Eq.()) at (-1,-10) {Eq.()};
\node (Eq.()) at (-1,-11) {Eq.()};
\draw[red] (Eq.(1)) -- (u0);
\draw[red] (Eq.(1)) -- (i0);
\draw[red] (Eq.(1)) -- (u1);
\draw[red] (Eq.(1)) -- (i1);
\draw[red] (Eq.(1)) -- (u2);
\draw[red] (Eq.(1)) -- (i2);
\draw[red] (Eq.(1)) -- (iC);
\draw[red] (Eq.(1)) -- (u4);
\draw[red] (Eq.(1)) -- (uL);
\draw[red] (Eq.(1)) -- (diL/dt);
\draw[red] (Eq.(1)) -- (du3/dt);
\draw[red] (Eq.(1)) -- (i4);
\end{tikzpicture}
\end{center}
[H7.1] Electrical Circuit, Horizontal and Vertical Sorting VI

\[ Eq.(1) \]
\[ Eq.(\ ) \]
\[ Eq.(\ ) \]
\[ Eq.(2) \]
\[ Eq.(\ ) \]
\[ Eq.(\ ) \]
\[ Eq.(\ ) \]
\[ Eq.(3) \]
\[ Eq.(\ ) \]
\[ Eq.(\ ) \]
\[ Eq.(\ ) \]
\[ Eq.(\ ) \]
\[ Eq.(\ ) \]
\[ Eq.(\ ) \]
\[ Eq.(\ ) \]
\[ Eq.(\ ) \]
\[ Eq.(\ ) \]
\[ Eq.(\ ) \]
\[ Eq.(\ ) \]
\[ Eq.(\ ) \]
[H7.1] Electrical Circuit, Horizontal and Vertical Sorting

\[
\begin{align*}
\text{Eq.}(1) & \quad \rightarrow \quad u_0 \\
\text{Eq.}(10) & \quad \rightarrow \quad i_0 \\
\text{Eq.}(11) & \quad \rightarrow \quad u_1 \\
\text{Eq.}(3) & \quad \rightarrow \quad i_1 \\
\text{Eq.}(12) & \quad \rightarrow \quad u_2 \\
\text{Eq.}(13) & \quad \rightarrow \quad u_L \\
\text{Eq.}(\) & \quad \rightarrow \quad diL/dt \\
\text{Eq.}(\) & \quad \rightarrow \quad du_3/dt \\
\text{Eq.}(\) & \quad \rightarrow \quad i_4
\end{align*}
\]
[H7.1] Electrical Circuit, Horizontal and Vertical Sorting

VII

\[ \text{Eq.}(1) \]
\[ \text{Eq.}(2) \]
\[ \text{Eq.}(3) \]
\[ \text{Eq.}(4) \]
\[ \text{Eq.}(5) \]
\[ \text{Eq.}(6) \]
\[ \text{Eq.}(7) \]
\[ \text{Eq.}(8) \]
\[ \text{Eq.}(9) \]
\[ \text{Eq.}(10) \]
\[ \text{Eq.}(11) \]
\[ \text{Eq.}(12) \]
\[ \text{Eq.}(13) \]
\[ \text{Eq.}(14) \]
\[ \text{Eq.}(15) \]

\( u_0 \)
\( i_0 \)
\( i_1 \)
\( u_1 \)
\( i_2 \)
\( u_2 \)
\( i_3 \)
\( i_C \)
\( u_4 \)
\( u_L \)
\( \frac{\text{di}_L}{\text{dt}} \)
\( \frac{\text{du}_3}{\text{dt}} \)
\( i_4 \)
[H7.1] Electrical Circuit, Horizontal and Vertical Sorting

VIII

\text{Eq.}(1)\quad u_0

\text{Eq.}(2)\quad i_0

\text{Eq.}(10)\quad i_1

\text{Eq.}(11)\quad u_1

\text{Eq.}(3)\quad i_2

\text{Eq.}(4)\quad u_2

\text{Eq.}(12)\quad i_3

\text{Eq.}(13)\quad u_3

\text{Eq.}(5)\quad i_4
[H7.1] Electrical Circuit, Horizontal and Vertical Sorting

VIII

\[ \begin{align*}
\text{Eq.}(1) & : u_0 \\
\text{Eq.}(2) & : u_1 \\
\text{Eq.}(3) & : \text{i}1 \\
\text{Eq.}(4) & : \text{i}2 \\
\text{Eq.}(5) & : \text{i}3 \\
\text{Eq.}(6) & : \text{i}4 \\
\text{Eq.}(7) & : \text{u}L \\
\text{Eq.}(8) & : \frac{\text{d}u_L}{\text{d}t} \\
\text{Eq.}(9) & : \frac{\text{d}u_3}{\text{d}t} \\
\end{align*} \]
[H7.1] Electrical Circuit, Horizontal and Vertical Sorting IX
[H7.1] Electrical Circuit, Horizontal and Vertical Sorting IX
[H7.1] Electrical Circuit, Horizontal and Vertical Sorting X
[H7.1] Electrical Circuit, Horizontal and Vertical Sorting X

1: \[ u_0 = 10 \]
6: \[ u_1 = R_1 \cdot i_1 \]
7: \[ u_2 = R_2 \cdot i_2 \]
2: \[ u_3 = R_3 \cdot i_3 \]
10: \[ i_C = C \cdot \frac{du_3}{dt} \]
11: \[ u_L = L \cdot \frac{di_L}{dt} \]
3: \[ i_4 = 4 \cdot u_3 \]
4: \[ u_0 = u_1 + u_3 \]
8: \[ u_L = u_1 + u_2 \]
12: \[ u_2 = u_3 + u_4 \]
13: \[ i_0 = i_1 + i_L \]
9: \[ i_1 = i_2 + i_C + i_3 \]
5: \[ i_4 = i_2 + i_L \]
[H7.1] Electrical Circuit, Horizontal and Vertical Sorting XI

1: \( u_0 = 10 \)
6: \( u_1 = R_1 \cdot i_1 \)
7: \( u_2 = R_2 \cdot i_2 \)
2: \( u_3 = R_3 \cdot i_3 \)
10: \( i_C = C \cdot \frac{du_3}{dt} \)
11: \( u_L = L \cdot \frac{di_L}{dt} \)
3: \( i_4 = 4 \cdot u_3 \)

4: \( u_0 = u_1 + u_3 \)
8: \( u_L = u_1 + u_2 \)
12: \( u_2 = u_3 + u_4 \)

13: \( i_0 = i_1 + i_L \)
9: \( i_1 = i_2 + i_C + i_3 \)
5: \( i_4 = i_2 + i_L \)
[H7.1] Electrical Circuit, Horizontal and Vertical Sorting XI

1: \( u_0 = 10 \)
6: \( u_1 = R_1 \cdot i_1 \)
7: \( u_2 = R_2 \cdot i_2 \)
2: \( u_3 = R_3 \cdot i_3 \)
10: \( i_C = C \cdot \frac{du_3}{dt} \)
11: \( u_L = L \cdot \frac{di_L}{dt} \)
3: \( i_4 = 4 \cdot u_3 \)
4: \( u_0 = u_1 + u_3 \)
8: \( u_L = u_1 + u_2 \)
12: \( u_2 = u_3 + u_4 \)

1: \( u_0 = 10 \)
2: \( i_3 = \frac{1}{R_3} \cdot u_3 \)
3: \( i_4 = 4 \cdot u_3 \)
4: \( u_1 = u_0 - u_3 \)
5: \( i_2 = i_4 - i_L \)
6: \( i_1 = \frac{1}{R_1} \cdot u_1 \)
7: \( u_2 = R_2 \cdot i_2 \)
8: \( u_L = u_1 + u_2 \)
9: \( i_C = i_1 - i_2 - i_3 \)
10: \( \frac{du_3}{dt} = \frac{1}{C} \cdot i_C \)
11: \( \frac{di_L}{dt} = \frac{1}{L} \cdot u_L \)
12: \( u_4 = u_2 - u_3 \)
13: \( i_0 = i_1 + i_L \)
The model and output equations can be coded as follows:

```matlab
function [xdot] = st_eq2(x, t)
R1 = 100;   R2 = 100;   R3 = 30;
C = 1e-6;   L = 0.01;

u3 = x(1);  iL = x(2);

u0 = 10;
i3 = u3/R3;
i4 = 4 * u3;
u1 = u0 - u3;
i2 = i4 - iL;
i1 = u1/R1;
u2 = R2 * i2;
uL = u1 + u2;
iC = i1 - i2 - i3;
du3 = iC/C;
diL = uL/L;
u4 = u2 - u3;
i0 = i1 + iL;

xdot = zeros(2, 1);
xdot(1) = du3;  xdot(2) = diL;

return
```
The model and output equations can be coded as follows:

```matlab
function [xdot] = st_eq2(x, t)
R1 = 100; R2 = 100; R3 = 30;
C = 1e-6; L = 0.01;

u0 = 10;
i0 = 0;
u3 = x(1); iL = x(2);

i3 = u3 / R3;
i4 = 4 * u3;
u1 = u0 - u3;
i2 = i4 - iL;
i1 = u1 / R1;
u2 = R2 * i2;
uL = u1 + u2;
iC = i1 - i2 - i3;

du3 = iC / C;
diL = uL / L;
u0 = u2 - u3;
i0 = i1 + iL;

xdot = zeros(2, 1);
xdot(1) = du3; xdot(2) = diL;

return
```

```matlab
function [y] = out_eq2(x, t)
R1 = 100; R2 = 100; R3 = 30;
C = 1e-6; L = 0.01;

u0 = 10;
i0 = 0;
u3 = x(1); iL = x(2);

i3 = u3 / R3;
i4 = 4 * u3;
u1 = u0 - u3;
i2 = i4 - iL;
i1 = u1 / R1;
u2 = R2 * i2;
uL = u1 + u2;
iC = i1 - i2 - i3;

du3 = iC / C;
diL = uL / L;
u0 = u2 - u3;
i0 = i1 + iL;

y = zeros(2, 1);
y(1) = u3; y(2) = iC;

return
```
The simulation loop (with Gustafsson step-size control) can be coded as follows:

```plaintext
while t < tf,
    [x4, x5] = rkf45_step2(x, t, h);
    err = norm(x4 - x5, 'inf') / max([norm(x4), norm(x5), 1.0e-10]);
    if err > tol,
        h = (0.8 * tol/err) * (0.2) * h;
        errl = 0;
    else
        t = t + h;
        x = x5;
        y = out_eq2(x, t);
        tvec = [tvec, t];
        yvec = [yvec, y];
        if errl > 0,
            h = (0.8 * tol/err) * (0.06) * (errl/err) * (0.08) * h;
        else
            h = (0.8 * tol/err) * (0.2) * h;
        end
        hvec = [hvec, h];
        errl = err;
    end
end
```
[H7.1] Electrical Circuit, Horizontal and Vertical Sorting

XIV

Homework [H7.1]

\[ Q_3 \]

\[ Q_2 \]

\[ h \]
[H7.7] Electrical Circuit, Structural Singularity

Given the circuit shown below containing three sinusoidal current sources:
Given the circuit shown below containing three sinusoidal current sources:
Write down the complete set of equations describing this circuit. Draw the structure digraph and begin causalizing the equations. Determine a constraint equation.
Given the circuit shown below containing three sinusoidal current sources:

- Write down the complete set of equations describing this circuit. Draw the structure digraph and begin causalizing the equations. Determine a constraint equation.

- Apply the Pantelides algorithm to reduce the perturbation index to 1. Then apply the tearing algorithm with substitution to bring the perturbation index down to 0.
[H7.7] Electrical Circuit, Structural Singularity

Given the circuit shown below containing three sinusoidal current sources:

- Write down the complete set of equations describing this circuit. Draw the structure digraph and begin causalizing the equations. Determine a constraint equation.

- Apply the Pantelides algorithm to reduce the perturbation index to 1. Then apply the tearing algorithm with substitution to bring the perturbation index down to 0.

- Write down the structure incidence matrices of the index-1 DAE and the index-0 ODE systems, and show that they are in BLT form, and in LT form, respectively.
[H7.7] Electrical Circuit, Structural Singularity II
[H7.7] Electrical Circuit, Structural Singularity II

\[\begin{align*}
1: \quad I_1 & = f_1(t) \\
2: \quad I_2 & = f_2(t) \\
3: \quad I_3 & = f_3(t) \\
4: \quad u_R & = R \cdot i_R \\
5: \quad i_C & = C \cdot \frac{du_C}{dt} \\
6: \quad u_{L1} & = L_1 \cdot \frac{di_{L1}}{dt} \\
7: \quad u_{L2} & = L_2 \cdot \frac{di_{L2}}{dt} \\
8: \quad u_1 & = u_{L1} + u_C \\
9: \quad u_{L1} & = u_R + u_2 \\
10: \quad u_{L2} & = u_2 + u_C \\
11: \quad u_3 & = u_R + u_{L2} \\
12: \quad I_1 + I_3 & = i_{L1} + i_R \\
13: \quad i_{L1} & = I_2 + i_C \\
14: \quad i_{L2} & = I_2 + i_R
\end{align*}\]
1: \( I_1 = f_1(t) \)  
2: \( I_2 = f_2(t) \)  
3: \( I_3 = f_3(t) \)  
4: \( u_R = R \cdot i_R \)  
5: \( i_C = C \cdot \frac{du_C}{dt} \)  
6: \( u_{L1} = L_1 \cdot \frac{dl_{L1}}{dt} \)  
7: \( u_{L2} = L_2 \cdot \frac{dl_{L2}}{dt} \)  
8: \( u_1 = u_{L1} + u_C \)  
9: \( u_{L1} = u_R + u_2 \)  
10: \( u_{L2} = u_2 + u_C \)  
11: \( u_3 = u_R + u_{L2} \)  
12: \( I_1 + I_3 = i_{L1} + i_R \)  
13: \( i_{L1} = I_2 + i_C \)  
14: \( i_{L2} = I_2 + i_R \)
1:  \[ I_1 = f_1(t) \]
2:  \[ I_2 = f_2(t) \]
3:  \[ I_3 = f_3(t) \]
4:  \[ u_R = R \cdot i_R \]
5:  \[ i_C = C \cdot \frac{du_C}{dt} \]
6:  \[ u_{L1} = L_1 \cdot \frac{di_{L1}}{dt} \]
7:  \[ u_{L2} = L_2 \cdot \frac{di_{L2}}{dt} \]
8:  \[ u_1 = u_{L1} + u_C \]
9:  \[ u_{L1} = u_R + u_2 \]
10: \[ u_{L2} = u_2 + u_C \]
11: \[ u_3 = u_R + u_{L2} \]
12: \[ I_1 + I_3 = i_{L1} + i_R \]
13: \[ i_{L1} = I_2 + i_C \]
14: \[ i_{L2} = I_2 + i_R \]
I causalized as much as I could without getting into trouble:

- Eq.(1)
- Eq.(2)
- Eq.(3)
- Eq.(8)
- Eq.(12)
- Eq.(11)
- Eq.(10)
- Eq.(14)
- Eq.(9)
- Eq.()
- Eq.(13)
- Eq.()
- Eq.(4)
- Eq.()
I causalized as much as I could without getting into trouble:

1: \( I_1 = f_1(t) \)
2: \( I_2 = f_2(t) \)
3: \( I_3 = f_3(t) \)
8: \( u_R = R \cdot i_R \)
12: \( i_C = C \cdot \frac{du_C}{dt} \)
11: \( u_{L1} = L_1 \cdot \frac{di_{L1}}{dt} \)
10: \( u_{L2} = L_2 \cdot \frac{di_{L2}}{dt} \)
14: \( u_1 = u_{L1} + u_C \)
9: \( u_{L1} = u_R + u_2 \)
?: \( u_{L2} = u_2 + u_C \)
13: \( u_3 = u_R + u_{L2} \)
?: \( I_1 + I_3 = i_{L1} + i_R \)
4: \( i_{L1} = I_2 + i_C \)
?: \( i_{L2} = I_2 + i_R \)
Any additional causalization leads invariably to a constraint:

\begin{align*}
\text{Eq.}(1) & \Rightarrow u_1 \\
\text{Eq.}(2) & \Rightarrow I_1 \\
\text{Eq.}(3) & \Rightarrow u_2 \\
\text{Eq.}(8) & \Rightarrow I_2 \\
\text{Eq.}(12) & \Rightarrow u_3 \\
\text{Eq.}(11) & \Rightarrow I_3 \\
\text{Eq.}(10) & \Rightarrow u_R \\
\text{Eq.}(14) & \Rightarrow i_R \\
\text{Eq.}(9) & \Rightarrow \frac{du_C}{dt} \\
\text{Eq.}(\cdot) & \Rightarrow i_C \\
\text{Eq.}(13) & \Rightarrow u_{L1} \\
\text{Eq.}(5) & \Rightarrow \frac{di_{L1}}{dt} \\
\text{Eq.}(4) & \Rightarrow u_{L2} \\
\text{Const.Eq.} & \Rightarrow \frac{di_{L2}}{dt}
\end{align*}
Any additional causalization leads invariably to a constraint:

1: \( I_1 = f_1(t) \)
2: \( I_2 = f_2(t) \)
3: \( I_3 = f_3(t) \)
8: \( u_R = R \cdot i_R \)
12: \( i_C = C \cdot \frac{du_C}{dt} \)
11: \( u_{L1} = L_1 \cdot \frac{di_{L1}}{dt} \)
10: \( u_{L2} = L_2 \cdot \frac{di_{L2}}{dt} \)
14: \( u_1 = u_{L1} + u_C \)
9: \( u_{L1} = u_R + u_2 \)
?: \( u_{L2} = u_2 + u_C \)
13: \( u_3 = u_R + u_{L2} \)
5: \( i_1 + i_3 = i_{L1} + i_R \)
4: \( i_{L1} = i_2 + i_C \)
const.eq.: \( i_{L2} = i_2 + i_R \)
We differentiate the constraint equation and let go of the integrator for \( i_{L1} \):

1. \( i_1 = f_1(t) \)
2. \( i_2 = f_2(t) \)
3. \( i_3 = f_3(t) \)
8. \( u_R = R \cdot i_R \)
12. \( i_C = C \cdot \frac{du_C}{dt} \)
11. \( u_{L1} = L_1 \cdot \frac{di_{L1}}{dt} \)
10. \( u_{L2} = L_2 \cdot \frac{di_{L2}}{dt} \)
14. \( u_1 = u_{L1} + u_C \)
9. \( u_{L1} = u_R + u_2 \)
? : \( u_{L2} = u_2 + u_C \)
13. \( u_3 = u_R + u_{L2} \)
5. \( i_1 + i_3 = i_{L1} + i_R \)
4. \( i_{L1} = i_2 + i_C \)
const.eq.: \( i_{L2} = i_2 + i_R \)
### [H7.7] Electrical Circuit, Structural Singularity VI

We differentiate the constraint equation and let go of the integrator for $i_{L1}$:

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<td>$\text{const.eq.}$</td>
<td>$i_{L2} = i_{L2} + i_R$</td>
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We introduced two new pseudo-derivatives, $dl_2$ and $di_R$:

1: $I_1 = f_1(t)$
2: $I_2 = f_2(t)$
3: $I_3 = f_3(t)$
10: $u_R = R \cdot i_R$
13: $i_C = C \cdot \frac{du_C}{dt}$
12: $u_{L1} = L_1 \cdot \frac{di_{L1}}{dt}$
?: $u_{L2} = L_2 \cdot di_{L2}$
15: $u_1 = u_{L1} + u_C$
11: $u_{L1} = u_R + u_2$
?: $u_{L2} = u_2 + u_C$
14: $u_3 = u_R + u_{L2}$
5: $I_1 + I_3 = i_{L1} + i_R$
4: $i_{L1} = l_2 + i_C$
6: $i_{L2} = l_2 + i_R$
?: $di_{L2} = dl_2 + di_R$
We introduced two new pseudo-derivatives, $dl_2$ and $di_R$:

1: $I_1 = f_1(t)$
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15: $i_C = C \cdot \frac{du_C}{dt}$
14: $u_{L2} = L_2 \cdot di_{L2}$
16: $u_3 = u_R + u_{L2}$
5: $I_1 + I_3 = i_{L1} + i_{R}$
4: $i_{L1} = I_2 + i_C$
6: $i_{L2} = I_2 + i_R$
?: $di_{L2} = dl_2 + di_R$
11: $u_{L1} = u_R + u_2$
17: $u_1 = u_{L1} + u_C$
?: $u_{L1} = u_R + u_2$
?: $u_{L2} = u_2 + u_C$
5: $I_1 + I_3 = i_{L1} + i_{R}$
4: $i_{L1} = I_2 + i_C$
6: $i_{L2} = I_2 + i_R$
?: $di_{L2} = dl_2 + di_R$
Two more pseudo-derivatives, $dl_1$ and $dl_3$:

1: $l_1 = f_1(t)$
2: $l_2 = f_2(t)$
7: $dl_2 = \frac{df_2(t)}{dt}$
3: $l_3 = f_3(t)$
8: $u_R = R \cdot i_R$
15: $i_C = C \cdot \frac{du_C}{dt}$
?: $u_{L1} = L_1 \cdot \frac{di_{L1}}{dt}$
?: $u_{L2} = L_2 \cdot di_{L2}$
17: $u_1 = u_{L1} + u_C$
?: $u_{L1} = u_R + u_2$
?: $u_{L2} = u_2 + u_C$
16: $u_3 = u_R + u_{L2}$
5: $l_1 + l_3 = i_{L1} + i_R$
?: $dl_1 + dl_3 = \frac{di_{L1}}{dt} + di_R$
4: $i_{L1} = l_2 + i_C$
6: $i_{L2} = l_2 + i_R$
?: $di_{L2} = dl_2 + di_R$
## Electrical Circuit, Structural Singularity VIII

Two more pseudo-derivatives, $dl_1$ and $dl_3$:

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We have an algebraic loop in six equations and six unknowns:

\[
\begin{align*}
?: & \quad u_{L1} \quad = \quad L_1 \cdot \frac{dL_1}{dt} \\
?: & \quad u_{L2} \quad = \quad L_2 \cdot dL_2 \\
?: & \quad u_{L1} \quad = \quad u_R + u_2 \\
?: & \quad u_{L2} \quad = \quad u_2 + u_C \\
?: & \quad dL_1 + dL_3 \quad = \quad \frac{dL_1}{dt} + dR \\
?: & \quad dL_2 \quad = \quad dL_2 + dR
\end{align*}
\]
We have an algebraic loop in six equations and six unknowns:

\[
\begin{align*}
? &: \quad u_{L1} &= L_1 \cdot \frac{d\imath_{L1}}{dt} \\
? &: \quad u_{L2} &= L_2 \cdot \imath_{L2} \\
? &: \quad u_{L1} &= u_R + u_2 \\
? &: \quad u_{L2} &= u_2 + u_C \\
? &: \quad \imath_{L1} + \imath_{L3} &= \frac{d\imath_{L1}}{dt} + \imath_{R} \\
? &: \quad \imath_{L2} &= \imath_{L2} + \imath_{R}
\end{align*}
\]
We have an algebraic loop in six equations and six unknowns:

\[ \begin{align*}
? &: \quad u_{L1} &= L_1 \cdot \frac{di_{L1}}{dt} \\
? &: \quad u_{L2} &= L_2 \cdot \frac{di_{L2}}{dt} \\
? &: \quad u_{L1} &= u_R + u_2 \\
? &: \quad u_{L2} &= u_2 + u_C \\
? &: \quad di_{L1} + di_3 &= \frac{di_{L1}}{dt} + di_R \\
? &: \quad di_{L2} &= di_2 + di_R
\end{align*} \]
We have an algebraic loop in six equations and six unknowns:

\[
\begin{align*}
? : & \quad u_{L1} = L_1 \cdot \frac{dL_1}{dt} \\
? : & \quad u_{L2} = L_2 \cdot \frac{dL_2}{dt} \\
? : & \quad u_{L1} = u_R + u_2 \\
? : & \quad u_{L2} = u_2 + u_C \\
? : & \quad dL_1 + dL_3 = \frac{dL_1}{dt} + dL_2 \\
? : & \quad dL_2 = dL_2 + dL_R
\end{align*}
\]

Res.Eq.

2: \quad u_{L1} = L_1 \cdot \frac{dL_1}{dt}
5: \quad u_{L2} = L_2 \cdot \frac{dL_2}{dt}
3: \quad u_{L1} = u_R + u_2
4: \quad u_{L2} = u_2 + u_C
1: \quad dL_1 + dL_3 = \frac{dL_1}{dt} + dL_2
res.eq.: \quad dL_2 = dL_2 + dL_R
We have an algebraic loop in six equations and six unknowns:

2: \[ u_{L1} = L_1 \cdot \frac{di_{L1}}{dt} \]
5: \[ u_{L2} = L_2 \cdot di_{L2} \]
3: \[ u_{L1} = u_R + u_2 \]
4: \[ u_{L2} = u_2 + u_C \]
1: \[ dl_1 + dl_3 = \frac{di_{L1}}{dt} + di_R \]
res.eq.: \[ di_{L2} = dl_2 + di_R \]
We have an algebraic loop in six equations and six unknowns:

\[
\begin{align*}
1: \quad dI_1 + dI_3 &= \frac{di_{L1}}{dt} + dI_R \\
\text{res.eq.:} \quad di_{L2} &= dl_2 + dI_R \\
\text{res.eq.:} \quad dI_R &= di_{L2} - dl_2 \\
2: \quad u_{L1} &= L_1 \cdot \frac{di_{L1}}{dt} \\
3: \quad u_{L1} &= u_R + u_2 \\
4: \quad u_{L2} &= u_2 + u_C \\
5: \quad di_{L2} &= \frac{1}{L_2} \cdot u_{L2}
\end{align*}
\]
We have an algebraic loop in six equations and six unknowns:

\[
\begin{align*}
2: \quad u_{L1} &= L_1 \cdot \frac{di_{L1}}{dt} \\
5: \quad u_{L2} &= L_2 \cdot di_{L2} \\
3: \quad u_{L1} &= u_R + u_2 \\
4: \quad u_{L2} &= u_2 + u_C \\
1: \quad dl_1 + dl_3 &= \frac{di_{L1}}{dt} + di_R \\
\text{res.eq.:} \quad di_{L2} &= dl_2 + di_R \\
\end{align*}
\]

\[
\begin{align*}
di_R &= di_{L2} - dl_2 \\
&= \frac{1}{L_2} \cdot u_{L2} - dl_2 \\
&= \frac{1}{L_2} \cdot u_2 + \frac{1}{L_2} \cdot u_C - dl_2 \\
&= \frac{1}{L_2} \cdot u_{L1} - \frac{1}{L_2} \cdot u_R + \frac{1}{L_2} \cdot u_C - dl_2 \\
&= \frac{L_1}{L_2} \cdot \frac{di_{L1}}{dt} - \frac{1}{L_2} \cdot u_R + \frac{1}{L_2} \cdot u_C - dl_2 \\
&= \frac{L_1}{L_2} \cdot dl_1 + \frac{L_1}{L_2} \cdot dl_3 - \frac{L_1}{L_2} \cdot di_R - \frac{1}{L_2} \cdot u_R + \frac{1}{L_2} \cdot u_C - dl_2
\end{align*}
\]
We have an algebraic loop in six equations and six unknowns:

\[\begin{align*}
2: \quad u_{L1} &= L_1 \cdot \frac{dl_1}{dt} \\
5: \quad u_{L2} &= L_2 \cdot dl_2 \\
3: \quad u_{L1} &= u_R + u_2 \\
4: \quad u_{L2} &= u_2 + u_C \\
1: \quad dl_1 + dl_3 &= \frac{di_{L1}}{dt} + dl_2 \\
\text{res.eq.:} \quad dl_2 &= dl_2 + dl_3
\end{align*}\]

\[\begin{align*}
\text{res.eq.:} \quad dl_3 &= dl_1 + dl_3 - dl_2 \\
2: \quad u_{L1} &= L_1 \cdot \frac{di_{L1}}{dt} \\
3: \quad u_2 &= u_{L1} - u_R \\
4: \quad u_{L2} &= u_2 + u_C \\
5: \quad dl_2 &= \frac{1}{L_2} \cdot L_1 \cdot dl_1 + L_1 \cdot dl_3 - \frac{L_1}{L_2} \cdot di_R - \frac{1}{L_2} \cdot u_R + \frac{1}{L_2} \cdot u_C - dl_2
\end{align*}\]

\[\begin{align*}
di_R &= \frac{1}{L_2} \cdot u_{L2} - dl_2 \\
&= \frac{1}{L_2} \cdot L_2 \cdot u_{L2} - L_2 \cdot dl_2 \\
&= \frac{1}{L_2} \cdot L_2 \cdot u_{L2} + \frac{1}{L_2} \cdot u_C - dl_2
\end{align*}\]

\[\begin{align*}
di_R &= \frac{1}{L_2} \cdot L_2 \cdot dl_1 + \frac{1}{L_2} \cdot L_1 \cdot dl_3 - \frac{L_1}{L_2} \cdot di_R - \frac{1}{L_2} \cdot u_R + \frac{1}{L_2} \cdot u_C - dl_2
\end{align*}\]

\[\begin{align*}
di_R &= \frac{L_1 \cdot (dl_1 + dl_3) - u_R + u_C - L_2 \cdot dl_2}{L_1 + L_2}
\end{align*}\]
1: \( I_1 = f_1(t) \)
2: \( I_2 = f_2(t) \)
3: \( I_3 = f_3(t) \)
4: \( i_C = i_{L1} - I_2 \)
5: \( i_R = I_1 + I_3 - i_{L1} \)
6: \( i_{L2} = I_2 + i_R \)
7: \( \frac{di_2}{dt} = \frac{df_2(t)}{dt} \)
8: \( u_R = R \cdot i_R \)
9: \( \frac{di_1}{dt} = \frac{df_1(t)}{dt} \)
10: \( \frac{di_3}{dt} = \frac{df_3(t)}{dt} \)
11: \( \frac{di_R}{dt} = \frac{L_1 \cdot (di_1 + di_3) - u_R + u_C - L_2 \cdot \frac{di_2}{dt}}{L_1 + L_2} \)
12: \( \frac{di_{L1}}{dt} = di_1 + di_3 - di_R \)
13: \( u_{L1} = L_1 \cdot \frac{di_{L1}}{dt} \)
14: \( u_2 = u_{L1} - u_R \)
15: \( u_{L2} = u_2 + u_C \)
16: \( \frac{di_{L2}}{dt} = \frac{1}{L_2} \cdot u_{L2} \)
17: \( \frac{du_C}{dt} = \frac{1}{C} \cdot i_C \)
18: \( u_3 = u_R + u_{L2} \)
19: \( u_1 = u_{L1} + u_C \)
[H7.7] Electrical Circuit, Structural Singularity XIII

\[ S = \]

\[
\begin{array}{cccccccccccccccc}
\text{l}_1 & \text{l}_2 & \text{l}_3 & \text{i}_C & \text{i}_R & \text{i}_L & \text{d}_L & \text{d}_1 & \text{d}_3 & \frac{\text{di}}{\text{dt}} & \text{u}_L & \text{u}_2 & \text{u}_L & \frac{\text{du}}{\text{dt}} & \text{u}_3 & \text{u}_1 \\
1: & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2: & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3: & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4: & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5: & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6: & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7: & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8: & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
9: & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
10: & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
11: & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
12: & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
13: & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
14: & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
15: & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
16: & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
17: & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
18: & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
19: & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
\end{array}
\]
The following set of DAEs:

\[
\begin{align*}
\frac{dC}{dt} &= K_1(C_0 - C) - R \\
\frac{dT}{dt} &= K_1(T_0 - T) + K_2R - K_3(T - T_C) \\
0 &= R - K_3 \exp \left( -\frac{K_4}{T} \right) C \\
0 &= C - u
\end{align*}
\]

describes a chemical isomerization reaction.
The following set of DAEs:

\[
\begin{align*}
\frac{dC}{dt} &= K_1(C_0 - C) - R \\
\frac{dT}{dt} &= K_1(T_0 - T) + K_2R - K_3(T - T_C) \\
0 &= R - K_3 \exp\left(\frac{-K_4}{T}\right) C \\
0 &= C - u
\end{align*}
\]

describes a chemical isomerization reaction.

*C* is the reactant concentration, *T* is the reactant temperature, and *R* is the reactant rate per unit volume. *C₀* is the feed reactant concentration, and *T₀* is the feed reactant temperature. *u* is the desired concentration, and *T_C* is the control temperature that we need to produce *u*. 
We want to turn the problem around (inverse model control) and determine the necessary control temperature $T_C$ as a function of the desired concentration $u$. Thus, $u$ will be an input to our model, and $T_C$ is the output.
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Draw the structure digraph. You shall notice at once that one of the equations has no connections to it. Thus, it is a constraint equation that needs to be differentiated, while an integrator associated with the constraint equation needs to be thrown out.
We want to turn the problem around (inverse model control) and determine the necessary control temperature $T_C$ as a function of the desired concentration $u$. Thus, $u$ will be an input to our model, and $T_C$ is the output.

Draw the structure digraph. You shall notice at once that one of the equations has no connections to it. Thus, it is a constraint equation that needs to be differentiated, while an integrator associated with the constraint equation needs to be thrown out.

We now have five equations in five unknowns. Draw the enhanced structure digraph, and start causalizing the equations. You shall notice that a second constraint equation appears. Hence the original DAE system had been an index-3 DAE system. Differentiate that constraint equation as well, and throw out the second integrator. In the process, new pseudo-derivatives are introduced that call for additional differentiations.
This time around, you end up with eight equations in eight unknowns. Draw the once more enhanced structure digraph, and causalize the equations. This is an example, in which (by accident) the Pantelides algorithm reduces the perturbation index in one step from 2 to 0, i.e., the final set of equations does not contain an algebraic loop.
This time around, you end up with eight equations in eight unknowns. Draw the once more enhanced structure digraph, and causalize the equations. This is an example, in which (by accident) the Pantelides algorithm reduces the perturbation index in one step from 2 to 0, i.e., the final set of equations does not contain an algebraic loop.

Draw a block diagram that shows how the output $T_C$ can be computed from the three inputs $u$, $\frac{du}{dt}$, and $\frac{d^2 u}{dt^2}$. 
The original equations are:

\[
\begin{align*}
? : & \quad \frac{dC}{dt} = K_1 \cdot (C_0 - C) - R \\
? : & \quad \frac{dT}{dt} = K_1 \cdot (T_0 - T) + K_2 \cdot R - K_3 \cdot (T - T_C) \\
? : & \quad 0 = R - K_3 \cdot \exp\left(\frac{-K_4}{T}\right) \cdot C \\
\text{const.eq.} : & \quad 0 = C - u
\end{align*}
\]
The original equations are:

\[ \frac{dC}{dt} = K_1 \cdot (C_0 - C) - R \]
\[ \frac{dT}{dt} = K_1 \cdot (T_0 - T) + K_2 \cdot R - K_3 \cdot (T - T_C) \]
\[ 0 = R - K_3 \cdot \exp\left(\frac{-K_4}{T}\right) \cdot C \]
\[ \text{const.eq.:} \quad 0 = C - u \]

With the structure digraph:
The original equations are:

\[ \begin{align*}
? & : \quad \frac{dC}{dt} = K_1 \cdot (C_0 - C) - R \\
? & : \quad \frac{dT}{dt} = K_1 \cdot (T_0 - T) + K_2 \cdot R - K_3 \cdot (T - T_C) \\
? & : \quad 0 = R - K_3 \cdot \exp\left(\frac{-K_4}{T}\right) \cdot C \\
\text{const.eq.:} & \quad 0 = C - u
\end{align*} \]

With the structure digraph:

![Structure digraph](image)

We recognize immediately a constraint equation.
The enhanced equations are:

\[
\begin{align*}
?: \quad dC &= K_1 \cdot (C_0 - C) - R \\
?: \quad \frac{dT}{dt} &= K_1 \cdot (T_0 - T) + K_2 \cdot R - K_3 \cdot (T - T_C) \\
?: \quad 0 &= R - K_3 \cdot \exp \left(\frac{-K_4}{T}\right) \cdot C \\
?: \quad 0 &= C - u \\
?: \quad 0 &= dC - \frac{du}{dt}
\end{align*}
\]
The enhanced equations are:

\[
\begin{align*}
? &: \quad \frac{dC}{dt} = K_1 \cdot (C_0 - C) - R \\
? &: \quad \frac{dT}{dt} = K_1 \cdot (T_0 - T) + K_2 \cdot R - K_3 \cdot (T - T_C) \\
? &: \quad 0 = R - K_3 \cdot \exp \left( -\frac{K_4}{T} \right) \cdot C \\
? &: \quad 0 = C - u \\
? &: \quad 0 = dC - \frac{du}{dt}
\end{align*}
\]

With the structure digraph:
We start coloring the structure digraph and recognize soon a second constraint equation:
[H7.8] Chemical Reactions, Pantelides Algorithm VI

We start coloring the structure digraph and recognize soon a second constraint equation:

\[
\begin{align*}
? : \quad dC &= K_1 \cdot (C_0 - C) - R \\
? : \quad \frac{dT}{dt} &= K_1 \cdot (T_0 - T) + K_2 \cdot R - K_3 \cdot (T - T_C) \\
? : \quad 0 &= R - K_3 \cdot \exp\left(-\frac{K_4}{T}\right) \cdot C \\
? : \quad 0 &= C - u \\
? : \quad 0 &= dC - \frac{du}{dt}
\end{align*}
\]
The once more enhanced equations are:

\[ \begin{align*}
?: \quad dC &= K_1 \cdot (C_0 - C) - R \\
?: \quad dT &= K_1 \cdot (T_0 - T) + K_2 \cdot R - K_3 \cdot (T - T_C) \\
?: \quad 0 &= R - K_3 \cdot \exp\left(\frac{-K_4}{T}\right) \cdot C \\
?: \quad 0 &= C - u \\
?: \quad 0 &= dC - \frac{du}{dt} \\
?: \quad d2C &= K_1 \cdot \left(\frac{dC_0}{dt} - dC\right) - dR \\
?: \quad 0 &= dR - K_3 \cdot \exp\left(\frac{-K_4}{T}\right) \cdot \left[ dC + \frac{K_4 \cdot C \cdot dT}{T^2} \right] \\
?: \quad 0 &= d2C - \frac{d^2u}{dt^2}
\end{align*} \]
Let us color the structure digraph:

![Diagram of chemical reactions and Pantelides Algorithm](image-url)
Let us color the structure digraph:

\[ \begin{align*}
&\text{Eq.}(1) \\
&\text{Eq.}(2) \\
&\text{Eq.}(3) \\
&\text{Eq.}(4) \\
&\text{Eq.}(5) \\
&\text{Eq.}(6) \\
&\text{Eq.}(7) \\
&\text{Eq.}(8)
\end{align*} \]

\[ \begin{align*}
T &\rightarrow \text{dT} \\
&\rightarrow \text{TC} \\
R &\rightarrow \text{dR} \\
C &\rightarrow \text{dC} \\
d2C &\rightarrow \\
\text{d2C} &\rightarrow \\
\end{align*} \]
Let us color the structure digraph:

We went from index-2 directly down to index-0. This sometimes happens.
4: \[ dC = K_1 \cdot (C_0 - C) - R \]
8: \[ dT = K_1 \cdot (T_0 - T) + K_2 \cdot R - K_3 \cdot (T - T_C) \]
6: \[ 0 = R - K_3 \cdot \exp\left(-\frac{K_4}{T}\right) \cdot C \]
1: \[ 0 = C - u \]
2: \[ 0 = dC - \frac{du}{dt} \]
5: \[ d2C = K_1 \cdot \left(\frac{dC_0}{dt} - dC\right) - dR \]
7: \[ 0 = dR - K_3 \cdot \exp\left(-\frac{K_4}{T}\right) \cdot \left[dC + \frac{K_4 \cdot C \cdot dT}{T^2}\right] \]
3: \[ 0 = d2C - \frac{d^2u}{dt^2} \]
1: \[ C = u \]
2: \[ dC = \frac{du}{dt} \]
3: \[ d^2C = \frac{d^2u}{dt^2} \]
4: \[ R = K_1 \cdot (C_0 - C) - dC \]
5: \[ dR = K_1 \cdot \left( \frac{dC_0}{dt} - dC \right) - d^2C \]
6: \[ T = \frac{-K_4}{\log\left(\frac{R}{K_3 \cdot C}\right)} \]
7: \[ dT = \frac{T^2}{K_3 \cdot K_4 \cdot C} \cdot \left[ dR \cdot \exp\left(\frac{K_4}{T}\right) - K_3 \cdot dC \right] \]
8: \[ TC = \frac{dT - K_1 \cdot (T_0 - T) - K_2 \cdot R + K_3 \cdot T}{K_3} \]
[H7.8] Chemical Reactions, Pantelides Algorithm XI